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Educational Television Series 1968-69

Teachers' Guide

Mathematics

Analysis SENIOR DIVISION

Educational Television Branch
ONTARIO DEPARTMENT OF EDUCATION
1670 Bayview Avenue, Toronto 17

MATHEMATICS
Senior Division

ANALYSIS

8 programs, 25 minutes each

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1670 Bayview Avenue,
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INTRODUCTION

This series of eight programs is intended for viewing by Grade 13 students, and is based on the course of study as outlined in Curriculum S 12C Unit 3 (Second Degree Relations in the Plane), and Unit 4 (Trigonometry). Program number 5, Trigonometry I (Introduction) could be used by a Grade 12 class as a review after the completion of the work on trigonometry. For each topic, it is not the intention of the programs to present all the material that is outlined in the course of study. The inclusion of particular topics in the series is not meant to indicate that it is any more important than any other topic that is not included.

The programs have been developed to complement and aid the work of the teacher and to incorporate mathematical models and animation to illustrate these topics. Where possible, simple practical situations applicable to the subject are described and related to the corresponding mathematical properties. Historical references are also made to the contributions of various individuals to the study of the conic sections and trigonometry.

In some cases, the programs may be used as a review of the mathematical topic while others may serve as an introduction to the subject.

Each program could be used as follows: one class period for the film, and subsequent discussion and one period allotted for post-viewing activities suggested after each program or supplemented by the questions obtained from the appropriate text books. It is essential that students be equipped with writing material to note any difficulties or points of interest during the program.

The host for the series is Mr. Morris Liebovitz, Southwood Secondary School, Galt, Ontario.

References

CONICS I, II, III, IV

1. SCHWARTY, ABRAHAM. — *Calculus and Analytic Geometry* — Holt, Rinehart and Winston.
2. KLINE, MORRIS. WESLEY, ADDISON. — *Mathematics for Liberal Arts.*

TRIGONOMETRY I, II, III, IV

1. SCHWARTY, ABRAHAM. — *Calculus and Analytic Geometry* — Holt, Rinehart and Winston.
2. KLINE, MORRIS. WESLEY, ADDISON. — *Mathematics for Liberal Arts.*
3. BRIEN, O. — *Sequences* π — Houghton Mifflin. (Thomas Nelson).
4. DAVIS. — *The Lore of Large Numbers* — Random House.

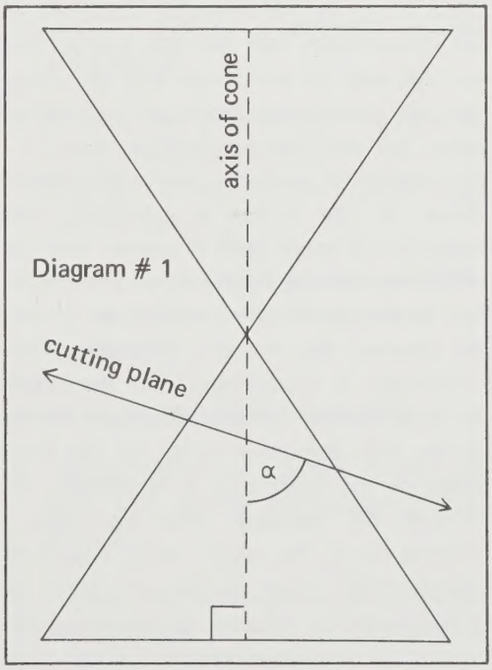
PROGRAM 1

CONICS I
FOCUS DIRECTRIX
ECCENTRICITY

The aims of the program are to introduce the focus directrix definitions of the ellipse, parabola, hyperbola and to relate them to the curve of intersection obtained by cutting a right circular cone by a plane.

PROGRAM CONTENT

The conics were investigated by the early Greek geometers but they studied them as an exercise in logic with little concern for their practical value. When a plane intersects a right circular cone, the curve of intersection is called a conic section, or conic. The plane meets the axis of the cone in the angle α, as seen in diagram number 1.



Animation is used to form the conics as the measurement of the angle varies. The question is raised whether the conic sections can be defined without requiring a three dimensional model of a cone. In the fourth century A.D., Pappus defined the conic sections in terms of a fixed point, called the focus, and a fixed line, called the directrix. He stated that the distance from a point P of the conic to the focus F is a constant multiple of its distance to the directrix. $|PF| = e|PD|$. Different values of "e" are used for animation and the corresponding conics are obtained:

$e < 1$ ellipse
$e = 1$ parabola
$e > 1$ hyperbola

The effective use of animation to develop these curves from the focus-directrix definitions is illustrated. The position of the focus and directrix with respect to the cone and a tangent sphere is then developed for the ellipse, parabola and hyperbola. If the cutting plane meets the axis of the right circular cone at a constant angle, the same conic is obtained but the size differs. The program closes with an examination of the degenerate conics and their relation to the cone.

SUGGESTED PRE-VIEWING ACTIVITIES

Familiarity with the terms cone, locus, sets of points, is required. The following questions could be used as a preparation for the telecast.

1. What are the curves of intersection when a plane meets a right circular cone?

2. Define a right circular cone and identify the nappe, vertex, generator, and axis.

3. What is the set of points in the plane for each of the following:

   (i) the points equidistant from two fixed points; three fixed points not in a straight line. For what condition is the set of points equidistant from four fixed points not null?

   (ii) The points equidistant from a fixed line; two intersecting fixed lines?

How would points in the plane equidistant from a fixed point, and a fixed line be determined?

4. The notation to indicate distance should be reviewed, e.g. if P, F, D are points then what does |PF|, |PD| mean?

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.

2. Discuss the contribution of Pappus and Menaechmus to the study of conics.

3. Instead of using a right circular cone intersected by a plane to obtain the conics, which conics can be obtained by using:

   (i) a right circular cylinder?

   (ii) an oblique circular cone?

4. For different values of the eccentricity, "e", determine the resulting conic section by the method used on the program.

5. Examine the meaning of the words focus, directrix, ellipse, parabola, hyperbola.

6. From the program, relate the position of the focus and directrix of the conics obtained by Pappus to the conics obtained when a plane meets a right circular cone. On the cone model, where is the directrix of the circle? As the angle of the cutting plane varies describe the resulting position of the focus and directrix.

7. What is meant by a degenerate conic? What is a degenerate ellipse, circle, parabola, hyperbola?

8. All parabolae have the same shape. Under what conditions, will a family of ellipses have the same shape when the cutting plane meets the right circular cone?

PROGRAM 2

CONICS II
FOCAL RADII

The aims of the program are to develop the defining equation of the ellipse and hyperbola with foci on the x-axis and to show the equivalence of the focus-directrix definitions and the constant sum and difference definitions for the ellipse and hyperbola respectively.

PROGRAM CONTENT

The program begins with a review of some of the concepts established in the previous program. The definitions of the conics ellipse, parabola, hyperbola, – are visually reviewed, and the terms focus, directrix, eccentricity related. A number of conics are obtained for specific values of the eccentricity. The focus-directrix definitions are related to the three-dimensional model, a right circular cone, and the conditions to obtain the conics are reviewed. In earlier studies of the circle, it was found that the simplest defining equation was obtained when the centre of the circle coincided with the origin of the cartesian plane. In a similar way, for each conic, the position of the coordinate axes must be placed so that the simplest defining equation of the conic is obtained. Animation is used to develop in detail the standard form of the defining equation of an ellipse with foci on the x-axis. The co-ordinates of the focus and the directrix are obtained in terms of the eccentricity "e" and the semi-major axis "a". By means of a ripple tank, the second focus of the ellipse is exhibited. The meaning of focal radii is shown and the definition of the ellipse due to Apollonius is developed by animation. In a similar manner, the defining equation of the hyperbola is introduced, and the ripple tank illustrates the existence of another focus. The definition based on the focal radii of the hyperbola is illustrated. Although the parabola does not have a second focus, the ripple tank is used to demonstrate some of the properties of this curve. The program closes with a reference to the development of the standard equation of a parabola, and the uses of its properties.

SUGGESTED PRE-VIEWING ACTIVITIES

The following could be used as a preparation for the telecast:

1. Draw the conics for the following values of the eccentricity $\frac{1}{2}$, $\frac{7}{8}$, 1, $\frac{5}{4}$.

2. Review the definition of a right circular cone, and the names of its parts.

3. Review how the conics are obtained when a plane intersects a right circular cone. What are the respective values of the eccentricity for the ellipse, parabola, hyperbola?

4. Develop the defining equation of a unit circle with the following centres: (i), (2,3), (ii), (2,0), (iii), (0,0). What simplifications are evident as the centres in (i), (ii), (iii), are used?

5. Simplify the following expression:
   $\sqrt{(x-ae)^2 + y^2} = e\sqrt{(x-\frac{a}{e})^2}$
   (This is essential to the program)

6. The notation for distance should be reviewed. If P, F_1, F_2, are points in the plane, state the meaning of
   $||PF_1| - |PF_2||$.

7. Discuss where the axes should be placed for the ellipse, parabola and hyperbola in order to obtain the simplest defining equation?

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.

2. For the development of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, in the program, answer the following questions:

   (a) State the co-ordinates of the focus in terms of "a" and "e". State the equation of the directrix in terms of "a" and "e".

   (b) Why is $b<a$?

   (c) State the x and y intercepts and the length of the major and minor axis in terms of "a" and "e".

Discuss the alternate definition of the ellipse due to Apollonius.

3. The ripple tank was used on the program to show the position of the other focus for the ellipse and for the hyperbola. Discuss the possible applications of this experiment in focusing sound and light waves.

4. Using the method developed on the program and the definition of the ellipse due to Apollonius, draw the ellipse with a = 8 and e = $\frac{7}{8}$ (or other suitable values.)

5. Complete the development of the defining equation of a hyperbola from the focus-directrix definitions.

6. Use the method developed on the program to draw the hyperbola when a = 4 and e = $\frac{5}{4}$.

7. If the focus of the parabola has co-ordinates (p,0), develop the defining equation of the parabola when (i) the vertex is at the origin (ii) the vertex is on the y-axis (iii) the vertex coincides with the point (a,b) where ab≠0.

PROGRAM 3

CONICS III

The aims of the program are to analyze by means of animation the curves defined by $\frac{x^2}{4} + \frac{y^2}{k^2} = 1$ *where k is any real number and to relate the resulting curve to the appropriate curve of intersection when a plane meets a right circular cone.*

PROGRAM CONTENT

Many properties of the conic sections were deduced by the Greeks from the curves of intersection obtained when a plane meets a right circular cone. Little did they realize the important role that these basic curves would play in science. The reflector properties are exhibited by a parabolic-elliptical pool table. The table was constructed by two students Stuart Hutcheson and Bob Christensen in London, Ontario. In program two the alternate definitions of the ellipse and hyperbola based on the constant sum and difference of the focal radii were introduced. In this program the two foci are related to the position of the curve of intersection obtained when a plane meets a right circular cone. From the model, the position of the other directrix is also shown. The property that the sum of the focal radii of an ellipse is a constant is deduced from the model of the cone and plane. The similar property for the hyperbola is also developed. By means of animation, the continuous change in the shape of the conics is seen as the value of eccentricity increases. The various conics obtained from the defining equation $\frac{x^2}{4} + \frac{y^2}{k^2} = 1$ is examined by animation for real values of k. In some cases the resulting curves are related to the corresponding curve obtained when a plane meets a right circular cone. The program closes with a further analysis of the other conics.

SUGGESTED PRE-VIEWING ACTIVITIES

1. Review the definitions of the conics from the focus directrix approach, and with respect to the sum and difference of the focal radii (ellipse and hyperbola).

2. Discuss how the reflector properties can be utilized in devices that require the focusing of energy.

3. Discuss what role, if any, the properties of the conics developed by the Greeks had in the study of science by Kepler and Newton.

4. Review from program one the position of the focus and directrix of the conics obtained when a plane meets a right circular cone.

5. Review the standard form of the defining equation of the circle, ellipse and hyperbola.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.

2. Discuss the questions in the pre-viewing activities in view of the material introduced on this program.

3. Determine the contribution of Kepler in the field of astronomy? What role did the conics play in his discoveries?

4. How is a parabolic-elliptical pool table constructed? What properties of the circle, parabola, ellipse, hyperbola could be utilized in the construction of similar tables?

5. Discuss the position of the directrix and the focus for each of the conic sections, when two spheres, as shown on the program, are placed in the cone.

6. If the curve of intersection is an ellipse when a plane meets a right circular cone, prove from the model that the sum of the focal radii of the ellipse is a constant.

7. Analyze the equation defined by $x^2 - y^2 = k$, k a real number. Examine the curves obtained for the special values k = 0, 1, -1.

PROGRAM 4

CONICS IV
APPLICATIONS

The aims of the program are to illustrate and describe examples of the occurrence of conic sections in applied mathematics and to relate the mathematical properties of the conics to the practical application considered.

PROGRAM CONTENT

The program deals with various present-day applications of the conics based on the properties of the conics that have been established in previous programs. The reflector properties of the ellipse, parabola, and hyperbola are utilized in many devices such as the headlights of cars, microwave reflectors, Jodrell Bank telescope. With some reflectors, sounds emanating from far distances can be focused and heard clearly. The telescope known as the Casgrain type makes use of the focusing properties of the parabola and hyperbola. The orbits of planets and

comets are conics and the conditions that determine an elliptical, parabolic, or hyperbolic path for these bodies are illustrated. Some satellites follow almost circular orbits about the earth while others follow elliptical paths. Terms such as *perigee* and *apogee* are introduced. The reasons why the paths of projectiles on the earth's surface approximate parabolae are noted, and the concept of minimum escape velocity is described. The uses of parabolic arcs in the American Space Program to obtain conditions of weightlessness and in the construction of highways are compared. The properties of the conic sections are used by man in many other ways: constructing bridges, erecting buildings. For example; a parabolic arc is used for a bridge not only because of the stresses involved but also because of its intrinsic beauty. The sail and the airplane wing are both designed to harness motion through the air, and their actual shapes are complicated curves. However, the properties of conic sections play an important role in their production. The program concludes with the development of a modern navigation system called LORAN. This system of determining the position of ships and airplanes depends on the constant difference property of the hyperbola.

SUGGESTED PRE-VIEWING ACTIVITIES

1. Review the following properties of the conics that have been established in earlier programs — the focus directrix definition of the ellipse, parabola, and hyperbola.— the reflector properties of the conics. — the curves obtained when the eccentricity has the following values: $e < 1$, $e = 1$, $e < 1$,
2. Review the meaning of the following terms with respect to each conic: intercepts, vertex, focus, directrix, focal radius.
3. Discuss the ripple tank experiments that exhibited the focusing properties of the ellipse, parabola, hyperbola, and determine practical situations which would require these properties.
4. What type of curves are used in the construction of highways, and cloverleaf junctions?
5. Establish situations in which the properties of the conic sections are utilized, e.g. the building of bridges with hyperbolic and parabolic spans, the domes of buildings in the form of elliptical arcs.

SUGGESTED POST-VIEWING ACTIVITIES

1. In the material that follows, some of the items were shown on the program. For each of the following determine which conic or conics are involved, and establish which property of the conic curve in question is utilized, if any. (These items could be assigned as material for seminar topics). No significance has been placed on the order, and the description has been kept to a minimum.
   - microwave systems
   - Casgrain telescope
   - Loran
   - headlights of cars, flashlights
   - orbits of satellites, and planets, comets
   - paths of projectiles
   - inverse square force field of repulsion or attraction
   - Kepler
   - Boyle's law
   - Jodrell Bank
   - beam of a flashlight on a wall
   - shape of highway curves
   - span of a bridge
   - gears
2. What is meant by conic lofting and how is it applied to the construction of aircraft?
3. Outline the contributions of Newton, and Kepler to the study of mathematics and physics as related to the properties of conic sections.

PROGRAM 5

TRIGONOMETRY I, INTRODUCTION

The aims of the program are to provide a review of the sine and cosine regarded as real valued functions of a real variable, and to indicate how the study of trigonometry from its earliest introduction based on angles, has been extended significantly into the analysis of periodic phenomena.

PROGRAM CONTENT

The program begins with a description of physical phenomena which are periodic. Trigonometry is referred to as a basis for the understanding of many mechanical devices such as radar, sonar, the hearing aid, and the speed trap. The history of trigonometry is traced from its use in the mensuration of the pyramids to its use as a computational tool and its extension into the study of periodic phenomena.

Two units of angular measurement (degree and radian) are obtained by means of a division of the circumference of a circle into equal arcs. By means of animation, the definitions of the three primary trigonometric functions are introduced.

The trigonometric function values are shown to be independent of the radius of the circle chosen in the definition thus allowing the unit circle to be used. The question is raised as to the meaning of $\frac{\pi}{2}$ in the expression $\sin \frac{\pi}{2}$. Since the number $\frac{\pi}{2}$ could be the measurement of an amount of rotation in radians or an arc length measured along the circumference of the unit circle, the θ in $\sin \theta$ is referred to as the *argument.* By means of animation, the graph of the function defined by $y = \sin \theta$ is drawn and its period, amplitude, maximum points and minimum points determined. The graph of the function defined by $y = \cos \theta$ is also animated and its period, amplitude, maximum points and minimum points determined. The curves defined by $y = \sin \theta$ or $y = \cos \theta$ are referred to as sinusoids.

An experiment is shown to describe how the motion of an oscillating body can draw a sinusoid. The motion of this body is then analysed by studying the resulting sinusoidal curve.

The program concludes with a reference to the defining equation of this sinusoid

and that a knowledge of the properties of the trigonometric functions is useful in analysing such periodic phenomena.

SUGGESTED PRE-VIEWING ACTIVITIES

Familiarity with the terms, arc length, ordinate, abscissa, standard position of an angle, function, real number and the equation π radians = 180 degrees is required in order to use the program to its fullest advantage. The following questions could be considered as a preparation for the telecast:

1. Discuss the uses of trigonometry. How is trigonometry used in computation?
2. Determine as many ways as possible of defining the sine function.
3. What are the differences and similarities between the curve defined by $y = \sin \theta$ and the curve defined by $y = \cos \theta$? For what reasons are they essentially the same?
4. What is x in the equation $y = \sin x$?
5. How could the unit circle be used in drawing the curve defined by $y = \sin \theta$?

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.
2. Discuss the advantages of using the term *argument* to describe x in the equations $y = \sin x$, $y = \cos x$.
3. Discuss why sinusoids are useful in the study of periodic phenomena.
4. List as many of the periodic phenomena referred to in the program and discuss the relation of trigonometry to them.

   How can the sinusoidal curve be related to these phenomena?
5. Set up an experiment similar to the one shown on the program to study the motion of an oscillating body. For example, a metal rod secured in a vise, pulled and released could be used to produce a sinusoidal curve.

PROGRAM 6

TRIGONOMETRY II: AMPLITUDE AND PERIOD

The aims of this program are to study the amplitude and period of a sinusoidal curve, to show their independence, and to consider applications of their properties.

PROGRAM CONTENT

The program begins by demonstrating how the oscilliscope can be used in the study of sound. When a tuning fork is struck, a sinusoidal curve is seen on the oscilliscope, thus indicating that the vibrating tuning fork and the vibrating strip (previous program) have the same characteristics; that is, both phenomena generate sinusoid.

A review of both the sine and cosine graphs is then given and reference is made to the five key points.

A tuning fork and the oscilliscope are used to illustrate the physical meaning of amplitude. The curves defined by equations such as $y = a \sin \theta$, a, real number, are then considered and their amplitudes are subsequently shown to be the absolute value of a.

Three tuning forks of different pitch are struck and the visual representation of these sounds on the oscilliscope provides an introduction to the period of sinusoids. The period of sinusoids with defining equations of the form $y = \sin K\theta$ are obtained by animation. If the number of cycles of the sinusoid that occur in the interval from 0 to 2π is $|K|$ then the period of $y = \sin K\theta$ is $\frac{2\pi}{|K|}$

A tuning fork, with an oscilliscope, is used to show that a change in the amplitude of a sinusoidal curve will not affect its period. It is then shown with tuning forks that a change in period of a sinusoidal curve will not affect its amplitude. The interdependence of amplitude and period is a fundamental requirement in radio broadcasting. An explanation of the meaning of amplitude modulation and frequency modulation is presented.

The equation $V = 120 \sin 120 \pi t$ which defines the oscillation of the voltage in an alternating current for each second of time "t" is then considered and the expressions peak voltage and 60 cycle circuits are introduced.

The program concludes with a reference to the relationship between periodic phenomena and trigonometry.

SUGGESTED PRE-VIEWING ACTIVITIES

Familiarity with the terms sinusoid, cycle, argument, defining equation, voltage, alternating current and the properties of the graph defined by $y = \sin \theta$ and $y = \cos \theta$ is required to use the program to its fullest advantage. The following question could be used as a preparation for the telecast:

1. Review the properties of the functions defined by $y = \sin \theta$, $y = \cos \theta$ developed in the previous program.
2. Discuss amplitude and period and their definitions with respect to periodic curves in general and sinusoids in particular.
3. How is a tuning fork and an oscilliscope used to obtain a visual description of sound waves?
4. What is the relationship of period and period of vibration.
5. Consider how the curve defined by $y = \cos 2\theta$ can be obtained from a unit circle.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.
2. Set up a tuning fork in conjunction with an oscilliscope and observe any other effects of striking the tuning fork.
3. Practice sketching curves of the form $y = a \sin K\theta$ using the 5-key points approach, for different values of a and K.
4. Discuss the effect of changing a and

K for the sinusoidal curve defined by $y = a \sin K\theta$.

5. Practice sketching curves of the form $y = a \cos K\theta$ using the 5-key points approach for different values of a and K.

6. Discuss the meaning of frequency modulation and amplitude modulation.

PROGRAM 7

TRIGONOMETRY III: PHASE SHIFT

The aims of this program are to introduce the notion of phase shift of a sinusoid with respect to a reference curve and to determine its defining equation and phase shift.

PROGRAM CONTENT

The program begins with a review of the properties of sinusoidal curves (amplitude and period) and how a study of them provides an insight into the nature of many scientific phenomena.

The notion of a phase shift of a sinusoid with reference to the curve defined by $y = \sin\theta$ is then introduced by examining the motion of a vibrating body for a positive, negative and zero phase shift. The cosine curve is shown to have a phase shift of $-\frac{\pi}{2}$ with reference to the curve defined by $y = \sin\theta$.

The meaning of phase shift with respect to the unit circle is shown in terms of animation. The defining equation of a sinusoidal curve for different phase shifts is then determined.

The sinusoids defined by $y = \sin(\theta + a)$ are determined, in detail with animation, for different values of "a".

The next section of the program shows how some basic identities for trigonometric functions can be established by examining sinusoidal curves with various phase shifts.

A number of oscillating bodies that are in simple harmonic motion is then shown and the equation determining simple harmonic motion is described as

$$y = a \sin(K\theta + d).$$

Sinusoids defined by equations of the form $y = a \sin(K\theta + d)$ are then sketched using the 5-key points approach and are shown to have a phase shift $-\frac{d}{K}$ with reference to the curve defined by

$$y = a \sin K\theta.$$

The program concludes with other examples of simple harmonic motion and reinforces the fact that a mathematical analysis of the equation $y = a \sin(K\theta + d)$ is necessary to understand these motions.

SUGGESTED PRE-VIEWING ACTIVITIES

Familiarity with the terms amplitude, period, sinusoid, 5-key points, argument, and the sketching of a sinusoidal curve from the unit circle is required to use the program to its fullest advantage. The following questions could be used as a preparation for the telecast.

1. Discuss how to plot a curve defined by $y = a \sin K\theta$ using the 5-key points approach.
2. Review the properties of amplitude and period established in the previous program.
3. Show that the sine and cosine curves are congruent.
4. Review and describe the motion of the vibrating body with respect to a sinusoidal curve.
5. List some examples of periodic phenomena that are studied, and require a knowledge of the properties of period and amplitude.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.

2. Review the steps in drawing the curve defined by the equation

$$y = 3 \sin(20 - \frac{\pi}{3})$$

using the 5-key points approach. Suggest other methods of sketching the sinusoid defined by this equation.

3. List the physical phenomena observed in the program that illustrate simple harmonic motion. Suggest other motions that are simple harmonic.

4. Discuss why the equation

$$y = a \sin(K\theta + d)$$

defined simple harmonic motion rather than the equation $y = a \sin K\theta$.

5. Practice sketching sinusoidal curves defined by $y = a \sin(K\theta + d)$ for arbitrary values of a, K and d.

6. Determine the phase shift for the following with respect to the appropriate reference curve:

i) $y = 3 \sin(\theta + \frac{\pi}{4})$

ii) $y = 2 \sin(3\theta - \pi)$

iii) $y = 2 \sin(2\theta + \frac{2}{\pi})$

iv) $y = 120 \cos(2\pi t + 3)$

v) $y = d \sin(a\theta + k)$

vi) $y = 4 \cos(\pi t - \frac{\pi}{2})$

PROGRAM 8

TRIGONOMETRY IV
a SIN θ + b COS θ

The aims of this program are to study the curves obtained by the synthesis of sinusoids and to analyze the curve defined by the equation $y = a \sin\theta + b \cos\theta$, $ab \neq 0$.

PROGRAM CONTENT

The program begins by illustrating with an oscilliscope how a complex curve can be formed by the addition of sinusoids. The striking of various combinations of tuning forks is used to duplicate musical sounds and the oscilliscope is used to illustrate their distinct patterns. Each oscilliscope pattern is obtained from the synthesis of sinusoids. Harmonic analysis is then referred to as a method used to analyze wave forms that are periodic and the dominant sinusoidal components of other wave forms.

Curves with defining equations of the form $y = a \sin \theta + b \cos \theta$ are then considered in detail. The curves resulting from this synthesis are shown to be sinusoidal with defining equations of the form $y = A \sin (\theta + d)$.

The relationships:

$$K = \sqrt{a^2 + b^2}$$

and $\tan d = \frac{b}{a}$

are then developed to simplify the sketching of curves defined by

$$y = a \sin \theta + b \cos \theta.$$

The study of synthesis is then extended to include three or more wave forms each with the same period. The synthesis of curves with different periods is then considered graphically. Reference is made to Joseph Fourier and his theory which states that the graph of any periodic function can be approximated by a sum of simple sinusoidal curves. Other tuning forks and the oscilliscope are then used to simulate the synthesis of many sinusoidal waves which are related to musical sound and the human voice.

The program concludes with a reference to the Fourier analysis of other phenomena in other apparently unrelated fields of physics.

SUGGESTED PRE-VIEWING ACTIVITIES

Familiarity with the terms phase shift, defining equation, sinusoid, pythagorean identities, and the expansion formulas such as $\sin (A+B) = \sin A \cos B + \cos A \sin B$ is required in order to use the program to its fullest advantage. The following questions could be used as a preparation for the telecast.

1. List some of the periodic phenomena that have been observed in the previous programs.

2. Discuss the use of a tuning fork and oscilliscope to produce a sinusoidal curve.

Discuss the amplitude, the period, the phase shift and the defining equation of the sinusoid obtained when more than one tuning fork are sounded.

SUGGESTED POST-VIEWING ACTIVITIES

1. Discuss points of interest in the program and clarify difficulties arising from it.

2. Discuss the relationship between the sounding of tuning forks and musical sounds.

3. Discuss the synthesis of sinusoids with the same period. In particular discuss the synthesis of curves with defining equations of the form $y = a \sin \theta$ and $y = b \cos \theta$, $ab \neq 0$.

4. Practice graphing the curve resulting from the synthesis of numerous sinusoids with different periods and amplitudes. Determine the periodic patterns and peak points.

5. Discuss whether or not the synthesis of any number of sinusoids will be periodic.

6. Discuss the uses of harmonic analysis.

7. Review the proof showing that:

$$K = \sqrt{a^2 + b^2}$$

and $\tan d = \frac{b}{a}$

when $a \sin \theta + b \cos \theta = K \sin (\theta + d)$.

8. Discuss the application of Fourier Analysis to study complex phenomena in heat transfer, building vibrations etc.

MATHEMATICS – Senior Division
ANALYSIS
8 programs, 25 minutes each

Prog. No.	CBC 11:00-11:25	CHCH 10:35-11:00	CKCO 9:30-9:55	CJOH 10:05-10:30	CKLW 10:00-10:25
1	OCT. 17 MAR. 28	OCT. 15 OCT. 17 MAR. 28	OCT. 25	JAN. 7	NOV. 1 MAR. 28 11:00-11:25
2	OCT. 24 APR. 11	OCT. 22 OCT. 24 APR. 11	NOV. 1	JAN. 14	NOV. 8 APR. 11 – 11:00-11:25
3	OCT. 31 APR. 18	OCT. 29 OCT. 31 APR. 18	NOV. 8	JAN. 21	NOV. 15 APR. 18 – 11:00-11:25
4	NOV. 7 NOV. 14 APR. 25 MAY 2	NOV. 5 NOV. 7 NOV. 12 NOV. 14 APR. 25 MAY 2	NOV. 15	JAN. 28	NOV. 22 APR. 25 – 11:00-11:25 MAY 2 – 11:00-11:25
5	NOV. 21 MAY 9	NOV. 19 NOV. 21 MAY 9	NOV. 22	FEB. 4	NOV. 29 MAY 9 – 11:00-11:25
6	NOV. 28 MAY 16	NOV. 25 NOV. 28 MAY 16	NOV. 29	FEB. 18	DEC. 6 MAY 16 – 11:00-11:25
7	DEC. 5 MAY 23	DEC. 3 DEC. 5 MAY 23	DEC. 6	FEB. 25	DEC. 13 MAY 23 – 11:00-11:25
8	DEC. 12 MAY 30	DEC. 10 DEC. 12 MAY 30	DEC. 13	MAR. 4	DEC. 20 JAN. 10 MAY 30 – 11:00-11:25

MATHEMATICS-ANALYSIS